Assignment 2

GCM 2010

Instructions. Answer the following, and be prepared to present and discuss your answers in class.

Preamble. Suppose that a test is ministered on two occasions to a group of people, and that the true scores of the individuals change from Time 1 to Time 2. We can model the scores on the two occasions as follows. $X_i = \rho_x^{1/2}\xi_i + (1-\rho_x)^{1/2}\epsilon_i = T_i + E_i$, $i = 1, 2$, with ρ_x the reliability of the test, ξ_i the standardized true score, T_i the true score, ϵ_i the standardized random error, and E_i the unstandardized error at time i. The ξ and ϵ components have variances of 1 and means of 0, the T_i and E_i have variances of ρ_x and $1-\rho_x$, respectively, and the errors are uncorrelated with each other and with the true scores. On the other hand, the true score components ξ_1 and ξ_2 are not uncorrelated. They have a correlation of ρ_{12}, as do T_1 and T_2. The reliability of the measure does not change from time 1 to time 2.

The observed change score is $D = X_1 - X_2$. The *true change score* is $D_t = T_1 - T_2$.

Using the theory of linear transformations and linear combinations, try to work the following problems.

1. Prove that X_1 and X_2 have variances of 1.

2. Write an expression for the variance of D in terms of ρ_{x_1,x_2}.

3. Write an expression for the variance of D_t in terms of ρ_{12} and ρ_x.

4. Prove that $\rho_{x_1,x_2} = \rho_x\rho_{12}$

5. In class, I gave the following expression for the reliability of the difference score:

$$\rho_D = \frac{\rho_x - \rho_{x_1,x_2}}{1 - \rho_{x_1,x_2}} \tag{1}$$

   where ρ_D is the reliability of D, ρ_x is the reliability of X_1 and of X_2, and ρ_{x_1,x_2} is the correlation between the observed scores at time 1 and time 2.

Beginning from the assumption that ρ_D is the proportion of variance of D that is true change score variance, i.e., that

$$\rho_D = \frac{\sigma^2_{D_t}}{\sigma^2_D}, \tag{2}$$

derive the formula I gave in class, using some of the other results you derived above.