Homework 2

Instructions. Feel free to email me for hints if you get stumped. Try to use R as much as possible, but remember that a numerical demonstration of something is not the same as an algebraic proof.

1. (40 points). Consider an m-factor common factor model, as presented in class. At the random variable level, the m-factor model (with all variables in deviation score form) states that

$$\boldsymbol{x} = \boldsymbol{\Lambda}\boldsymbol{\xi} + \boldsymbol{\delta} \tag{1}$$

with

$$\mathrm{E}(\boldsymbol{\xi}\boldsymbol{\xi}') = \boldsymbol{\Psi}, \quad \mathrm{E}(\boldsymbol{\xi}\boldsymbol{\delta}') = \mathbf{0}, \quad \mathrm{E}(\boldsymbol{\delta}\boldsymbol{\delta}') = \boldsymbol{U}^2 \tag{2}$$

where $\boldsymbol{U}^2$ is a diagonal, positive-definite matrix, $\boldsymbol{\Lambda}$ is the *common factor pattern*, $\boldsymbol{\Psi}$ the *factor intercorrelation matrix*, and $\boldsymbol{U}^2$ contains the *unique variances* of the variables on its diagonal. If $\boldsymbol{\Psi}$ is an identity matrix and the factors are uncorrelated, we say that the solution is *orthogonal*, otherwise it is *oblique.*

   (a) Using expected value algebra, prove the "Fundamental Theorem of Factor Analysis," i.e., that

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Psi}\boldsymbol{\Lambda}' + \boldsymbol{U}^2 \tag{3}$$

   (b) Suppose the factors are orthogonal, i.e., $\boldsymbol{\Psi} = \boldsymbol{I}$, an identity matrix. Find a simple expression for $\mathrm{Cor}(\boldsymbol{x}, \boldsymbol{\xi})$, i.e., the matrix of correlations between the observed variables in $\boldsymbol{x}$ and the common factors in $\boldsymbol{\xi}$. (Strong Hint: The factors and observed variables both are in standard score form, which simplifies your task considerably.)

2. Download the handout *Advanced EFA in R* and work through the example. Then analyze the `Thurstone` data set.

   (a) Examine the sequential chi-square test, the sequential difference test, the scree test, the RMSEA, and choose a number of factors.

   (b) Examine the patterns produced by various rotational criteria and select a rotation.

   (c) Describe your interpretation of the resulting factors.