Homework 4

Psychology 313

Instructions. Show your R code, your input, and your output. Feel free to email me for hints if you get stumped.

1. (20 points.) Suppose that the set of scores in the $n \times 3$ matrix $\mathbf{Y}$ has a sample variance-covariance matrix of

$$\mathbf{S}_{yy} = \begin{bmatrix} 1.05 & 0.58 & 0.45 \\ 0.58 & 2.07 & 0.88 \\ 0.45 & 0.88 & 1.92 \end{bmatrix} \tag{1}$$

   (a) What is the correlation matrix $\mathbf{R}_{yy}$ corresponding to $\mathbf{S}_{yy}$?

   (b) Suppose you create two new variables as linear combinations of the variables in $\mathbf{Y}$. They are created as

$$\mathbf{W} = \mathbf{YB} \tag{2}$$

   with

$$\mathbf{B} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} \tag{3}$$

   Find the 2×2 covariance matrix $\mathbf{S}_{ww}$ for the new variables in $\mathbf{W}$.

2. (10 points.) Suppose you have 6 raw scores in a vector $\mathbf{x}$. What linear combination of those 6 scores will compute the deviation score corresponding to x_2, the second score in $\mathbf{x}$? (Hint from lecture. Compute $\mathbf{Q}$, the deviation score projector, and examine its rows.)

3. (20 points.) Suppose you isolate the last row and column of a $p \times p$ *symmetric* matrix $\mathbf{A}$ by partitioning it into a "2×2 partitioned form" as follows

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{X} & \mathbf{b} \\ \hline \mathbf{b}' & a_{p,p} \end{array} \right] \tag{4}$$

where $a_{p,p}$ is the lower right element of $\mathbf{A}$. A well-known result is that if A has an inverse, it may be calculated from the inverse of its upper left submatrix $\mathbf{X}$ as

$$\mathbf{A}^{-1} = \frac{1}{k} \left[\begin{array}{c|c} k\mathbf{X}^{-1} + \mathbf{X}^{-1}\mathbf{b}\mathbf{b}'\mathbf{X}^{-1} & -\mathbf{X}^{-1}\mathbf{b} \\ \hline -\mathbf{b}'\mathbf{X}^{-1} & 1 \end{array} \right] = \frac{1}{k}\mathbf{W} \tag{5}$$

where $k = a_{p,p} - \mathbf{b}'\mathbf{X}^{-1}\mathbf{b}$.

Exercise your skills at partitioned matrix algebra by proving that the result is correct.

(Hints. Take the direct route by simply multiplying the partitioned form for matrix $\mathbf{A}$ in Equation 4 by the partitioned form for $\mathbf{A}^{-1}$ shown in Equation 5 and showing that it reduces to a 2×2 partitioned form that is, in fact, an identity matrix $\mathbf{I}$. Watch out for expressions of the general form $\mathbf{b}'\mathbf{X}\mathbf{b}$ or $\mathbf{b}'\mathbf{b}$ that are scalars. Break the problem down into smaller parts, i.e., remember that the result is a 2×2 partitioned form, and that the result must me symmetric, so if you can prove that the upper right part of the partitioned form is a null vector, you have automatically proven that the lower left is also a null vector. So you really need process only 3 submatrices of the 2×2 result. Work slowly and methodically. You may find that things are less confusing if you keep k out of the picture by first multiplying $\mathbf{A}$ by the partitioned form $\mathbf{W}$ and then bringing k in at the very end.)